

Exercise 1. The exclusive or, denoted by \oplus , outputs true when one, but not both, of its inputs are true. One way to write \oplus in terms of the basic symbols is

$$p \oplus q = (p \vee q) \wedge \neg(p \wedge q).$$

Use a truth table to show that $(p \wedge \neg q) \vee (q \wedge \neg p)$ is an equivalent way to write $p \oplus q$.

Proof. To show that the two are equivalent expressions with a truth table, we need to show that their columns have identical entries.

p	q	$p \vee q$	$\neg(p \wedge q)$	$(p \vee q) \wedge \neg(p \wedge q)$	$p \wedge \neg q$	$q \wedge \neg p$	$(p \wedge \neg q) \vee (q \wedge \neg p)$
T	T	T	F	F	F	F	F
T	F	T	T	T	T	F	T
F	T	T	T	T	F	T	T
F	F	F	T	F	F	F	F

Since the columns for both $(p \vee q) \wedge \neg(p \wedge q)$ and $(p \wedge \neg q) \vee (q \wedge \neg p)$ are $FTTTF$, the statements are logically equivalent, i.e., $(p \vee q) \wedge \neg(p \wedge q) \equiv (p \wedge \neg q) \vee (q \wedge \neg p)$. \square

Fun Math Facts

- (1) $\frac{22}{7}$ is a common fraction used to approximate the value of π .
- (2) $\frac{22}{7}$ is a bigger fraction than $\frac{22}{7}$.
- (3) The equation $e^{i\pi} + 1 = 0$ is known as the most beautiful equation in mathematics.
- (4) $\sqrt{2}$ and $\sqrt[3]{3}$ are irrational numbers.
- (5) One of DeMorgan's laws for sets is $(A \cap B)^C = A^C \cup B^C$.
- (6) $\pi \in \mathbb{R}$ but $\pi \notin \mathbb{Q}$.
- (7) A function, f , from a set A to a set B is denoted by $f : A \rightarrow B$.
- (8) A really interesting sum is $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

Math 220 - Discrete Mathematics

Homework 1 - Due Friday, September 7th

For this assignment, your goal is to replicate the reverse side of this page. You will need to turn in two things: hand in a printout of the compiled pdf (with “Your Name Here” replaced with your name) and submit on Canvas the .tex file with your code. This is (likely) the only assignment where I will require you to submit your .tex file.